



V, I, R measurements: how to generate and measure quantities and then how to get data (resistivity, magnetoresistance, Hall).

590B

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September 25, 2009

Resistivity

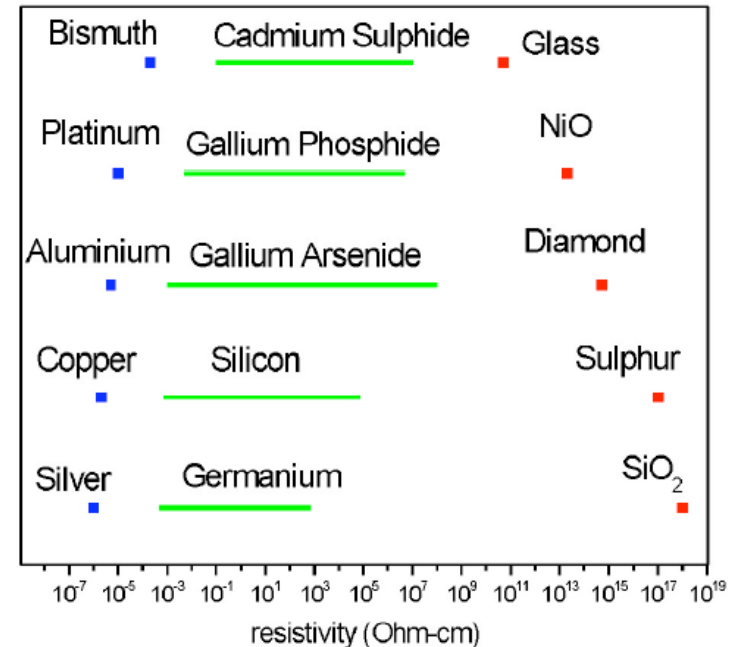
**Typical resistivity temperature dependence:
metals, semiconductors**

Magnetic scattering



Resistivities of Real Materials

Material	Resistivity 20°C
Silver	$1.59 \mu\Omega \text{ cm}$
Copper	$1.72 \mu\Omega \text{ cm}$
Aluminum	$2.82 \mu\Omega \text{ cm}$
Tungsten	$5.8 \mu\Omega \text{ cm}$
Platinum	$11 \mu\Omega \text{ cm}$
Manganin	$48 \mu\Omega \text{ cm}$
Constantan	$49 \mu\Omega \text{ cm}$
Nichrome	$110 \mu\Omega \text{ cm}$
Germanium	4.6
Glass	$10^8\text{-}10^{12} \Omega \text{ cm}$
PET	$10^{18} \Omega \text{ cm}$
Teflon	$10^{20}\text{-}10^{22} \Omega \text{ cm}$



Most semiconductors in their pure form are not good conductors, they need to be doped to become conducting.

Not all so called “ionic” materials like oxides are insulators.



➤ We can relate the conductivity, σ , of a material to microscopic parameters that describe the motion of the electrons (or other charge carrying particles such as holes or ions).

$$\text{➤ } \sigma = ne(e\tau/m^*)$$

$$\text{➤ } \mu = e \tau / m^*$$

$$\text{➤ } \sigma = ne \mu$$

➤ where

➤ n = the carrier concentration (cm^{-3})

➤ e = the charge of an electron = 1.602×10^{-19} C

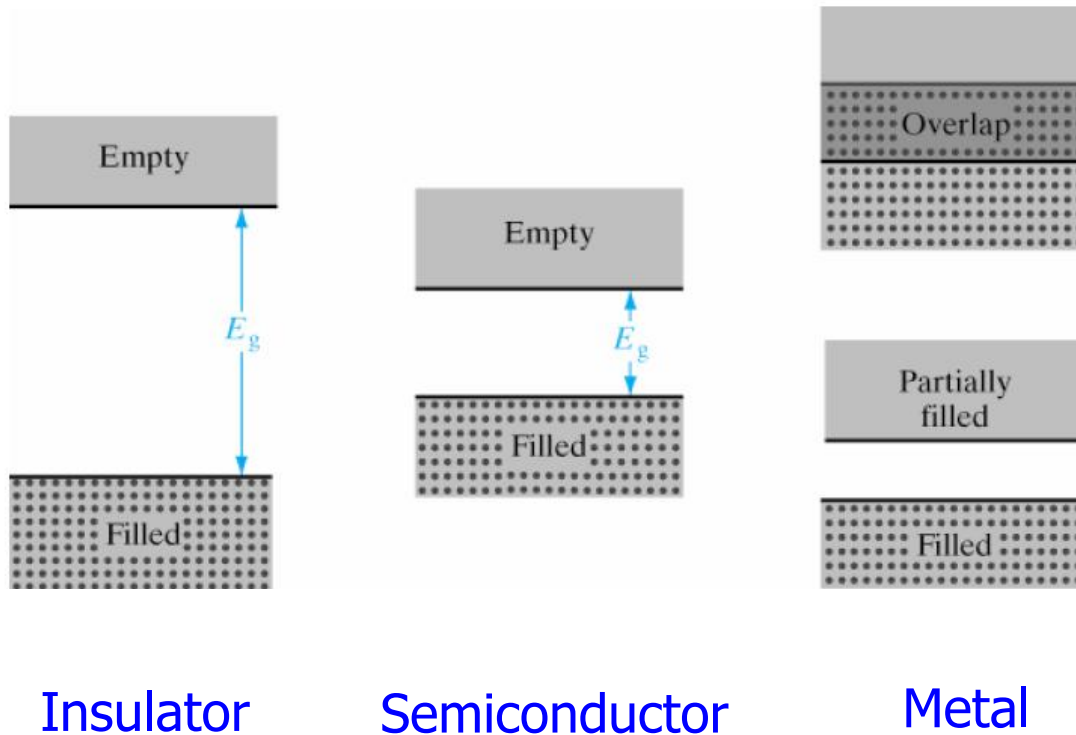
➤ τ = the relaxation time (s) {the time between collisions}

➤ m^* = the effective mass of the electron (kg)

➤ μ = the electron mobility ($\text{cm}^2/\text{V-s}$)



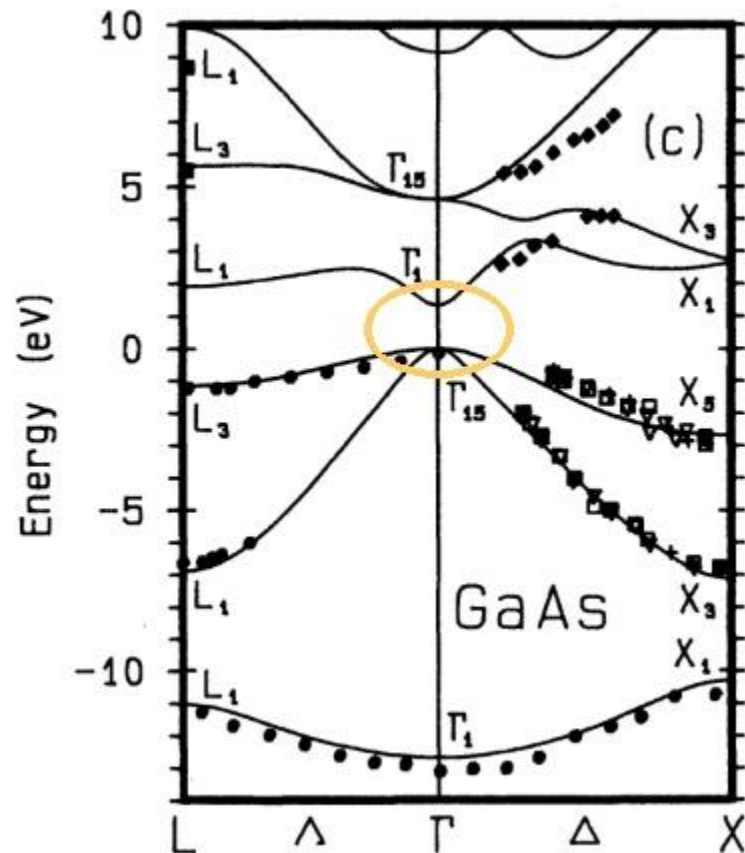
Metals, Semiconductors & Insulators



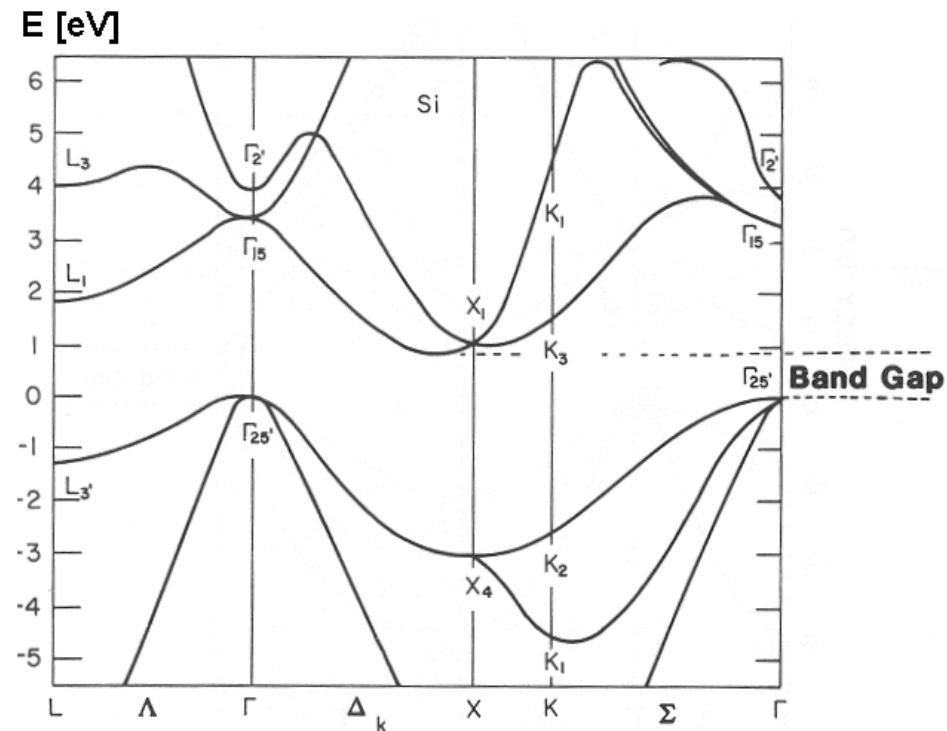
Big variation is due to big difference in free carrier density



Energy bands in semiconductors



Direct-gap
III-V
II-VI



Indirect band-gap
Si, Ge



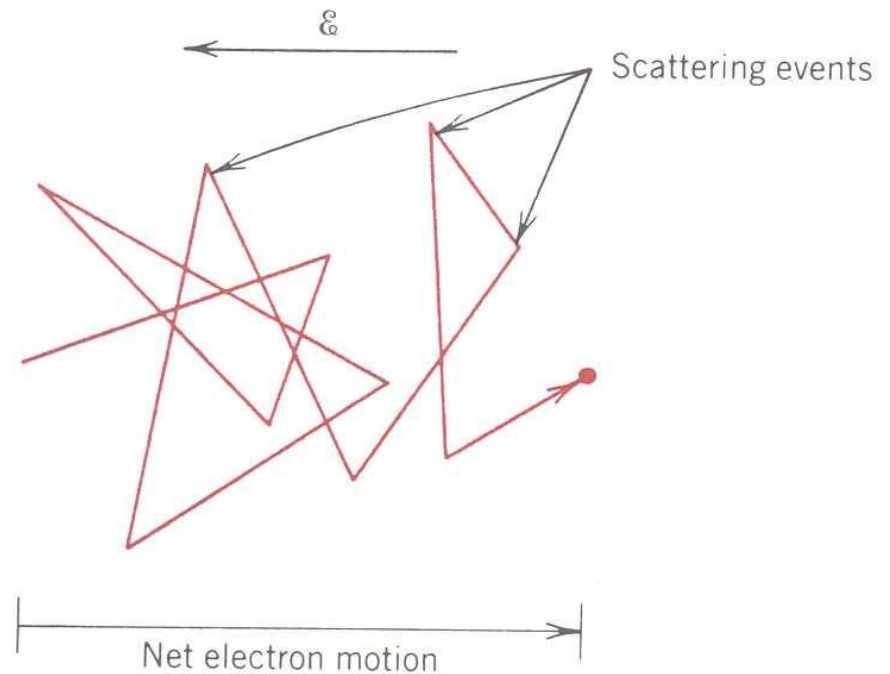
Compound	Structure	Bandgap (eV)	e ⁻ mobility (cm ² /V-s)	h ⁺ mobility (cm ² /V-s)
Si	Diamond	1.11 (I)	1,350	480
Ge	Diamond	0.67 (I)	3,900	1,900
AlP	Sphalerite	2.43 (I)	80	---
GaAs	Sphalerite	1.43 (D)	8,500	400
InSb	Sphalerite	0.18 (D)	100,000	1,700
AlAs	Sphalerite	2.16 (I)	1,000	180
GaN	Wurtzite	3.4 (D)	300	---

Electron mobility and scattering

What scatters carriers?

Disruptions in periodicity

- defects
- lattice vibrations
- surfaces





- **Thermal energy** \Rightarrow some electrons are excited from the valence band into the conduction band \Rightarrow **conduction electrons**
- This leaves an empty state in the valence band — a **hole**
 - n : density of CE
 - p : density of holes

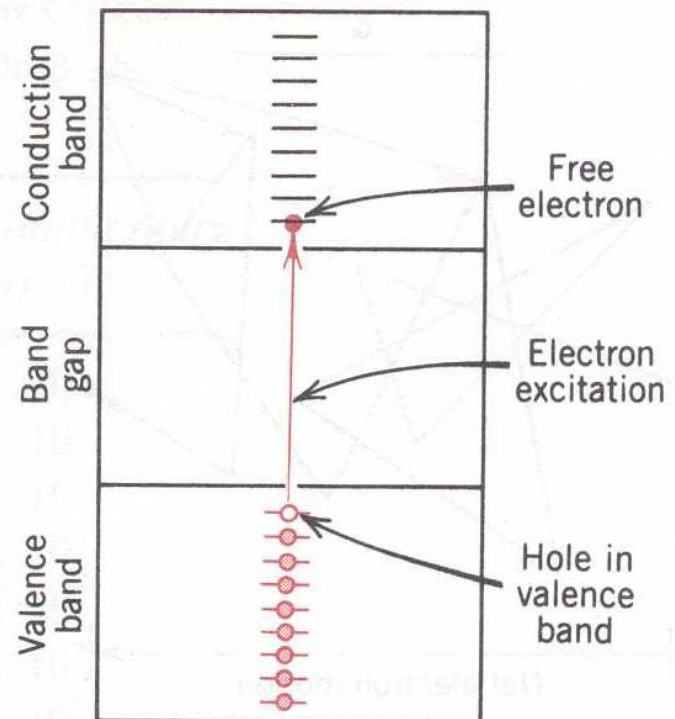
$$n \cdot p = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right)$$

- Intrinsic semiconductor: electrons and holes form in pairs

$$n_i = p_i = \sqrt{N_C N_V} \exp\left(-\frac{E_g}{2k_B T}\right)$$

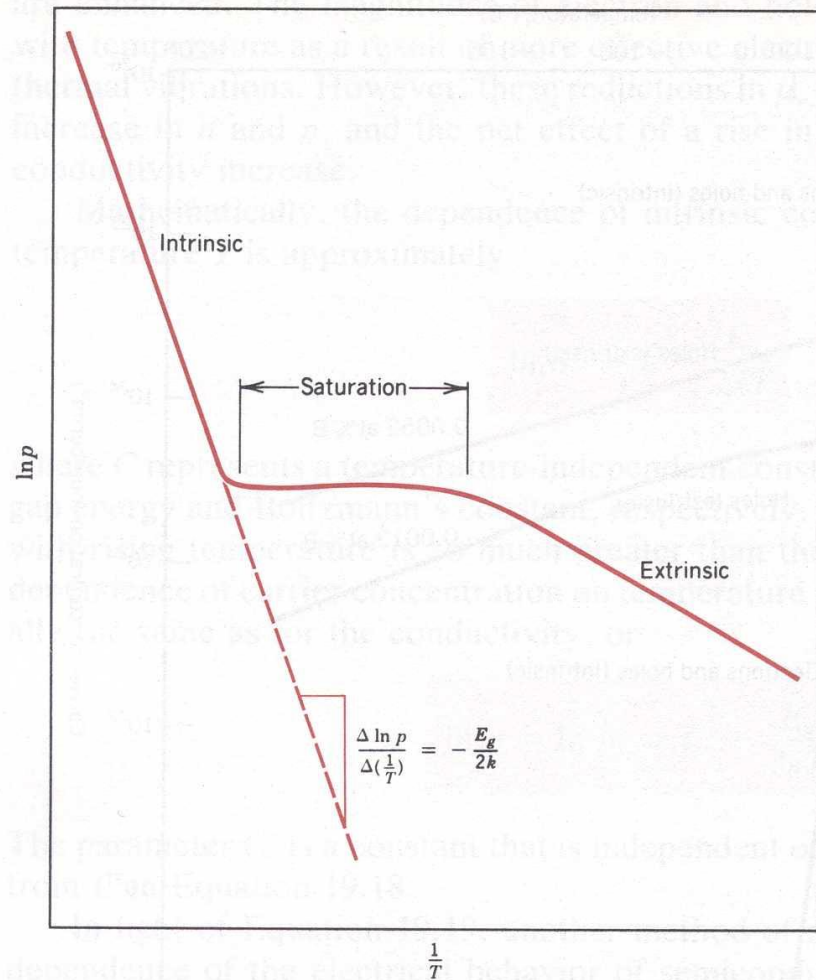
- Conductivity:

$$\sigma = n_i e \mu_e + p_i e \mu_h = n_i e (\mu_e + \mu_h)$$



Carrier density

- Low T: **ionization** of dopants
 - *n*-type: donor atoms donate e^- to conduction band
 - *p*-type: acceptor atoms accept e^- from valence band
- Intermediate T: all dopants ionized
 - *n*-type: **exhaustion** of donors
 - *p*-type: **saturation** of acceptors
- High T: intrinsic behavior



Schematic plot of the natural logarithm of hole concentration as a function of the reciprocal of absolute temperature for a *p*-type semiconductor that exhibits extrinsic, saturation, and intrinsic behavior.



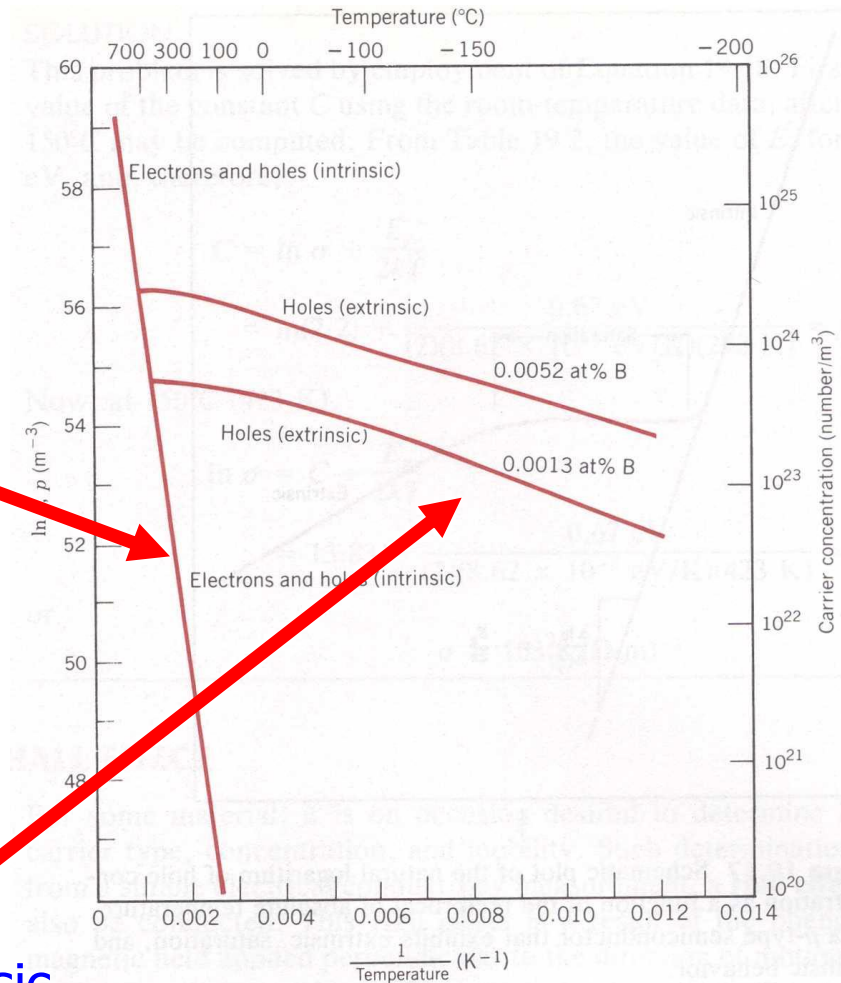
Can obtain bandgap for intrinsic region

$$n = p = \sqrt{N_C N_V} \exp\left(-\frac{E_g}{2k_B T}\right)$$

Intrinsic

If density of dopants $N \gg n, p$
Behaves like extrinsic conductor

Extrinsic



The logarithm of carrier (electron and hole) concentration as a function of the reciprocal of the absolute temperature for intrinsic silicon and two boron-doped silicon materials. (Adapted from G. L. Pearson and J. Bardeen, *Phys. Rev.*, **75**, 865, 1949.)

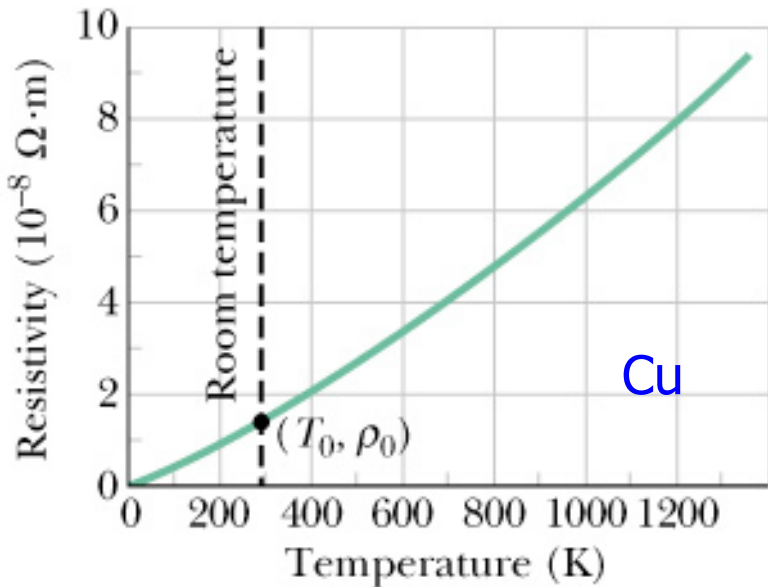
$$\sigma = ne\mu_e + pe\mu_h \cong ne\mu_e = N_d e\mu_e$$

Metals

D. MATTHIESSEN'S RULE

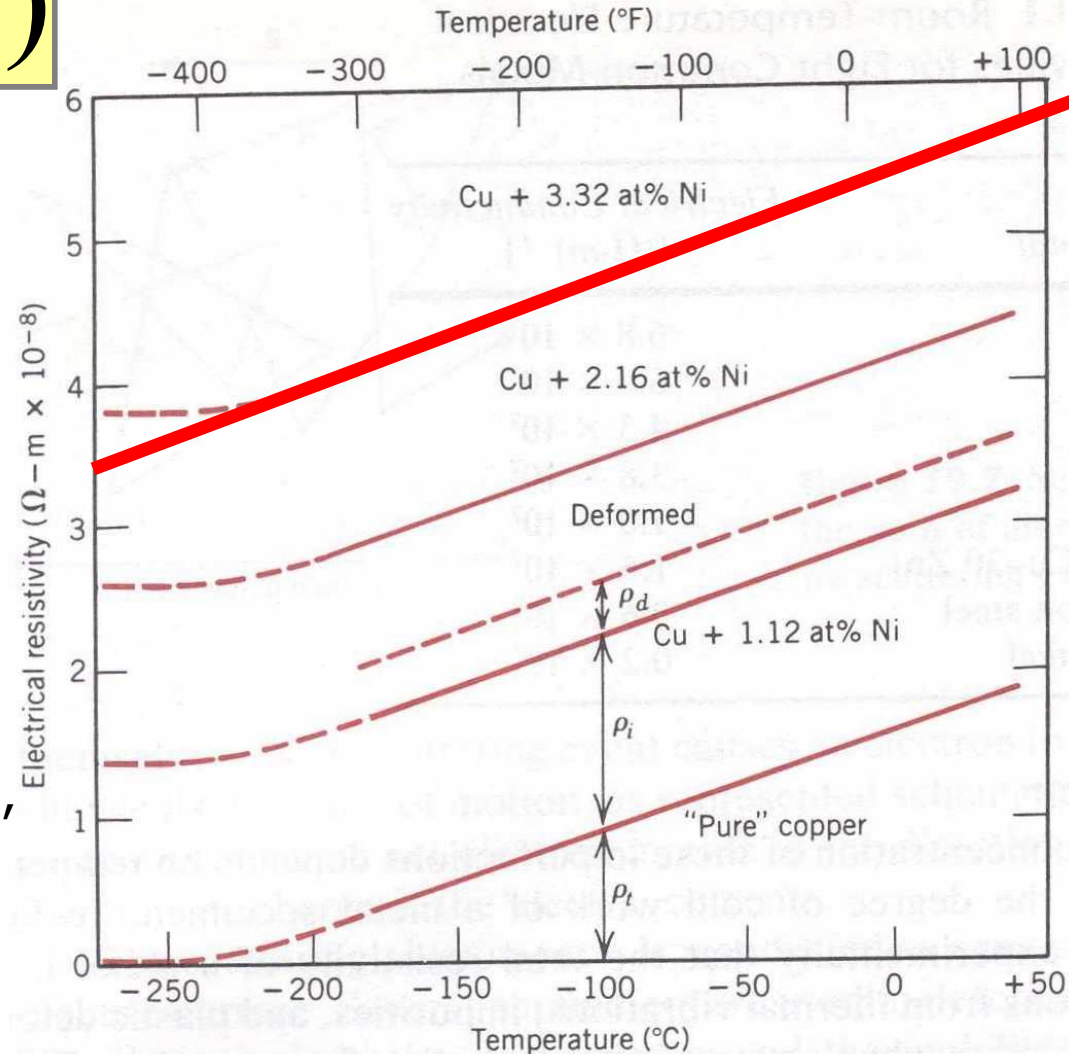
$$R = R_0 + R_T,$$

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$



Temperature coefficient of resistivity α reflects electron-phonon interaction, property of material

- Thermal vibrations scatter carriers
- \Rightarrow mobility \downarrow
- \Rightarrow conductivity \downarrow
- \Rightarrow resistivity \uparrow





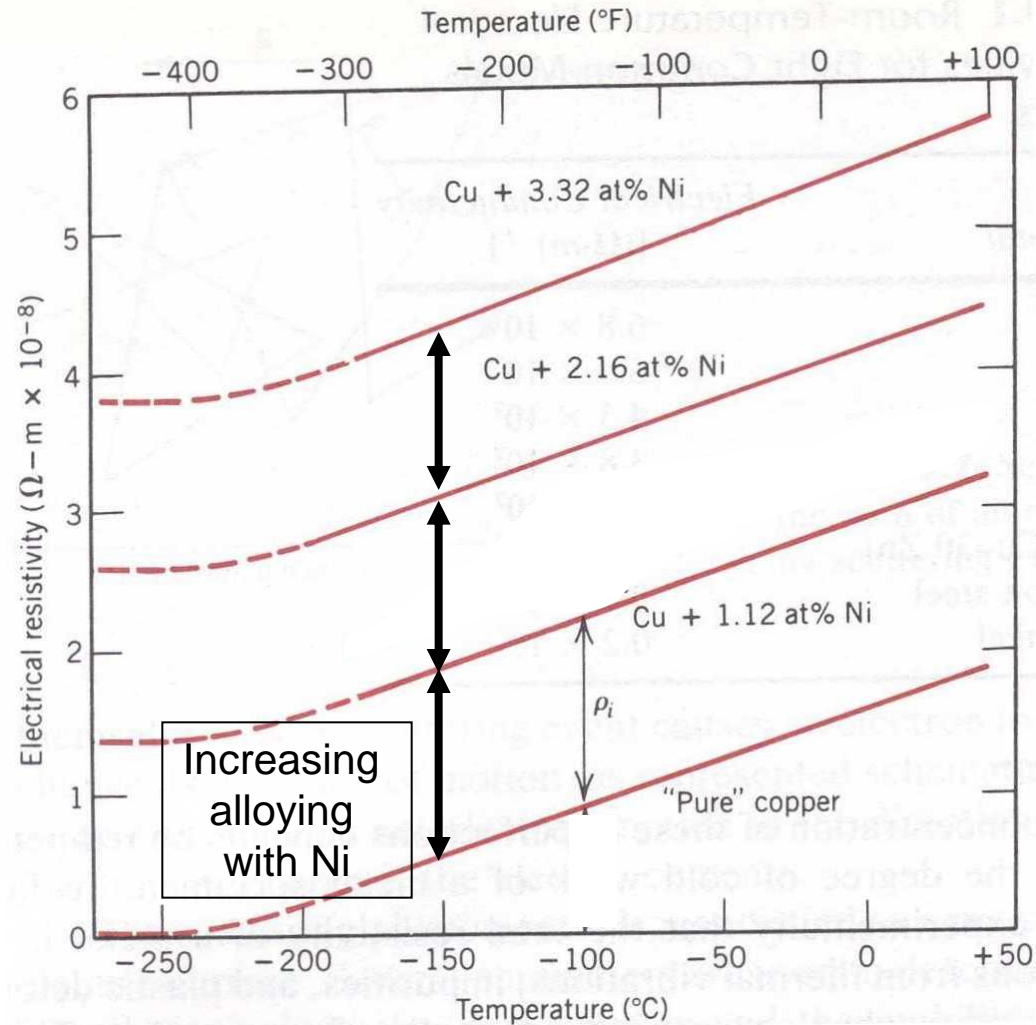
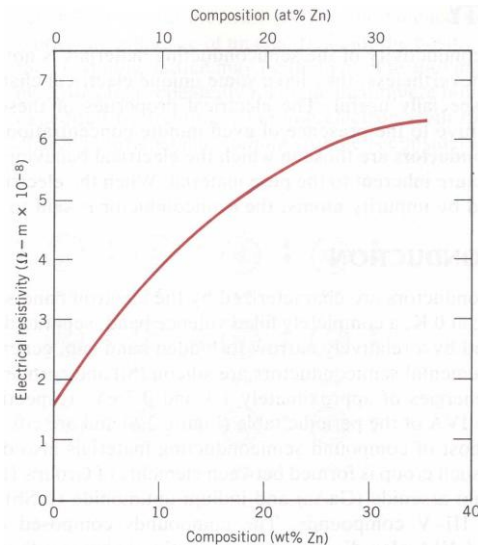
D. MATTHIESSEN'S RULE

$$R = R_0 + R_T,$$

➤ Effect of **impurities** on μ

$$\rho_i = A c_i (1 - c_i)$$

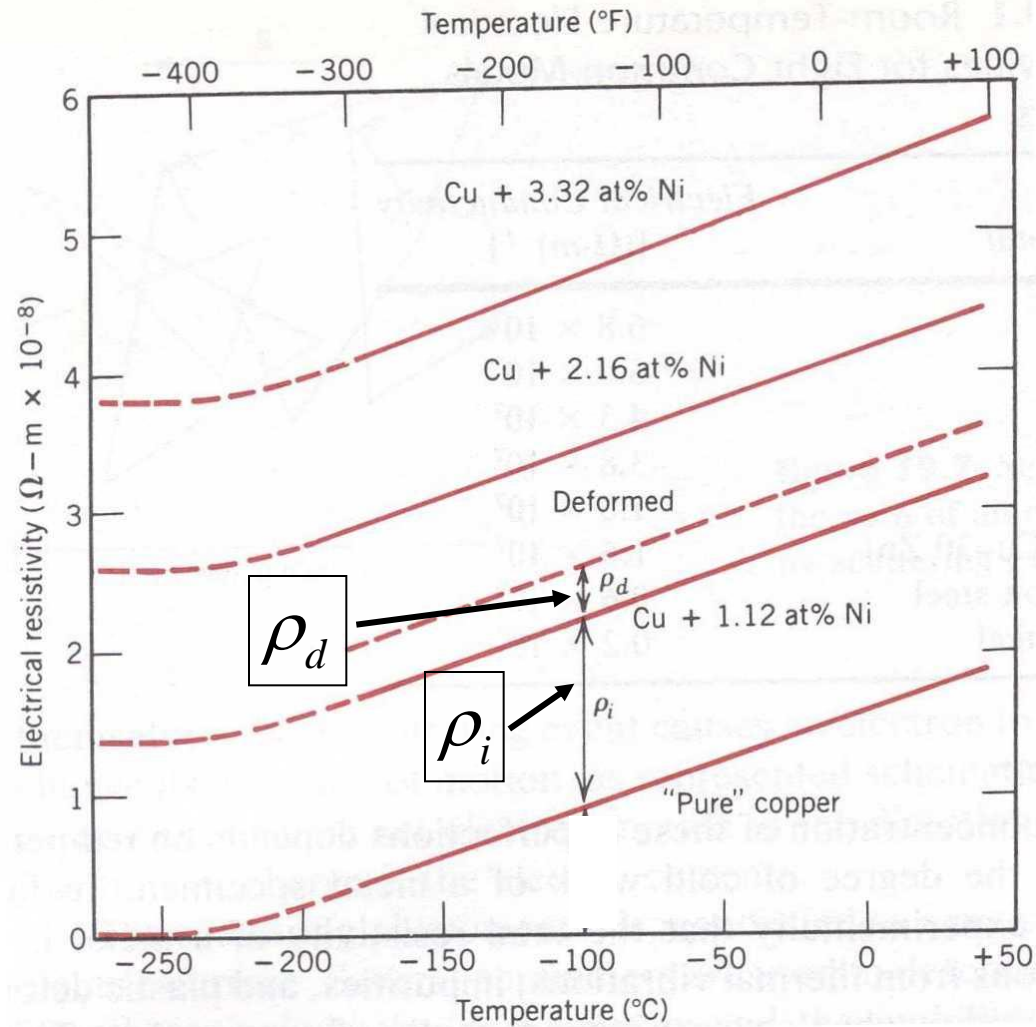
- A : property of host and impurity
- c_i : atomic fraction of impurity





➤ Effect of **deformation** on μ

- Dislocations (regions of deformed material) scatter carriers, increasing the electrical resistivity by ρ_D



Resistivity temperature dependence of metals closer look

High temperature

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$

$T \gg \Theta_D$ density of phonons $\sim T$

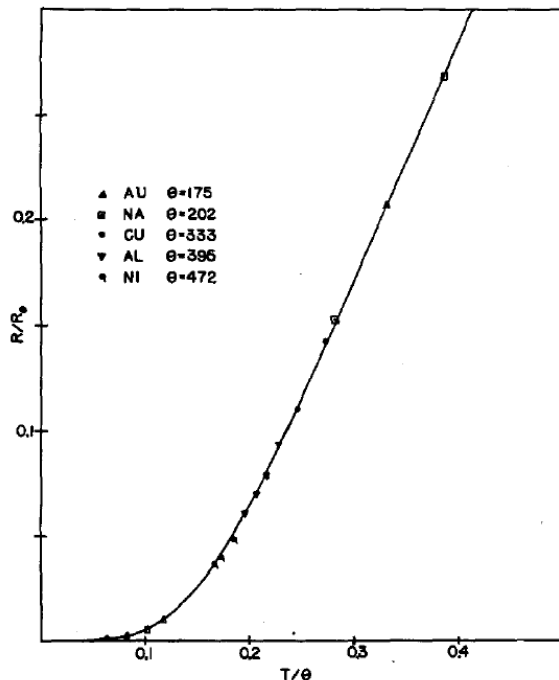
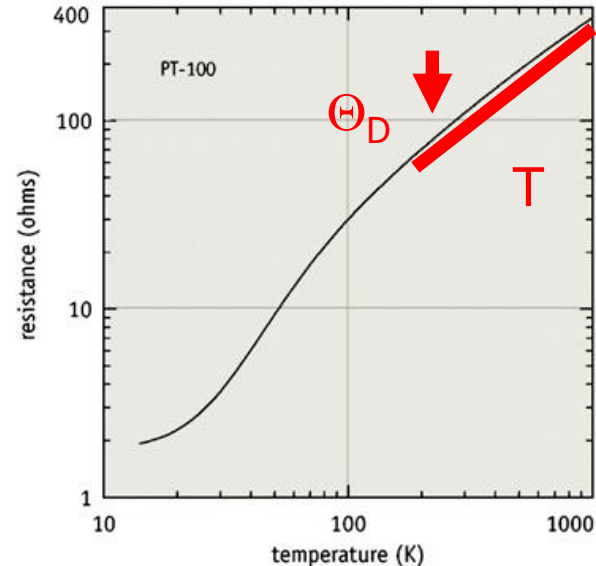


FIG. 1. Temperature variation of resistance of various metals. The curve is a plot of the Bloch-Grüneisen function (Eq. (28)). Data from values quoted by Meissner (cf. Bibliography).

$T < \Theta_D$ spectrum of phonons changes

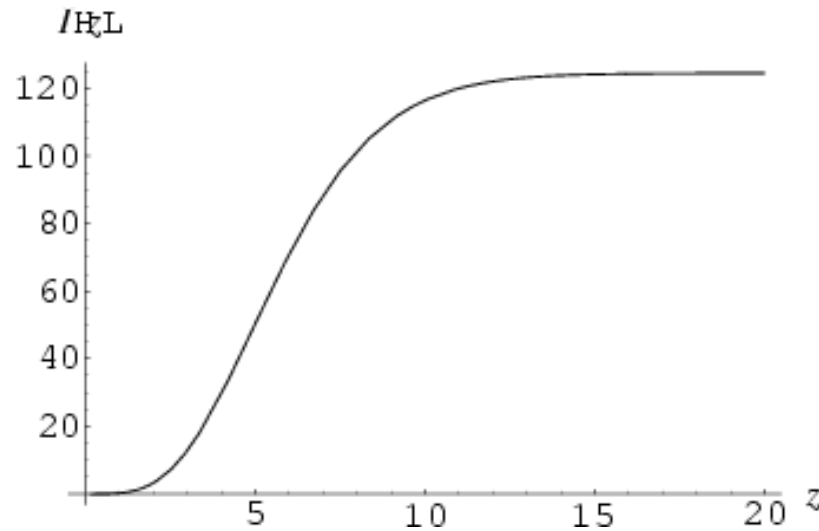
Bloch-Grüneisen law

For most of metals

$$R = R_0 f(T/\Theta),$$

Θ is close to Debye temperature in specific heat, not the same due to selectivity of scattering

$$\rho(T) = \rho(0) + A \left(\frac{T}{\Theta_R} \right)^n \int_0^{\frac{\Theta_R}{T}} \frac{x^n}{(e^x - 1)(1 - e^{-x})} dx$$



xT

$$\rho_{ph} \sim f(q) S_{ph}$$

With some
“filtering” due to
q-dependence of
scattering probability

Comparison of Θ_R and Θ_D

Metal	Ag	Au	Al	Cu	Pd	Pt	Rh
Θ_R	200	200	395	320	270	240	340
Θ_D	220	185	385	320	300	225	350

After G. T. Meaden Electrical Resistance of Metals

$q=0$ at $T=0$

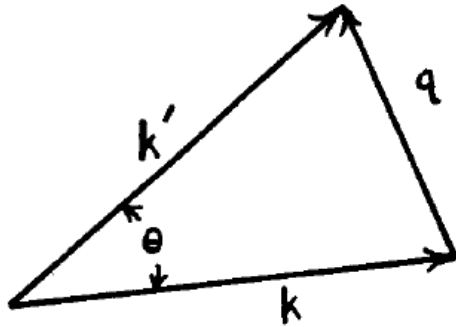


FIG. 11. Illustrating the vector relations for the scattering of an electron wave by a lattice wave.

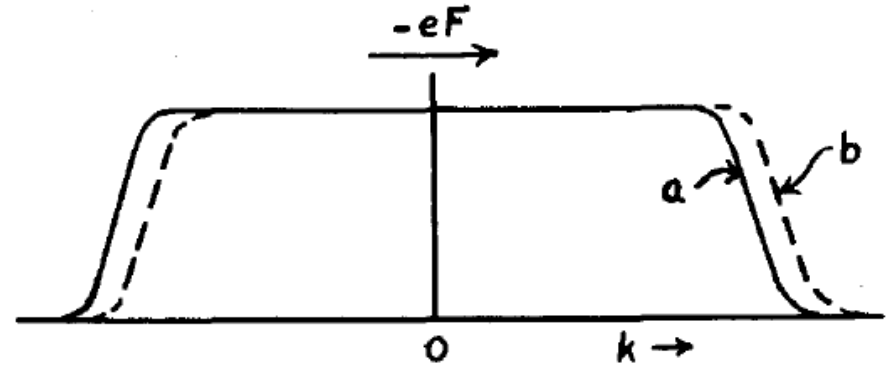


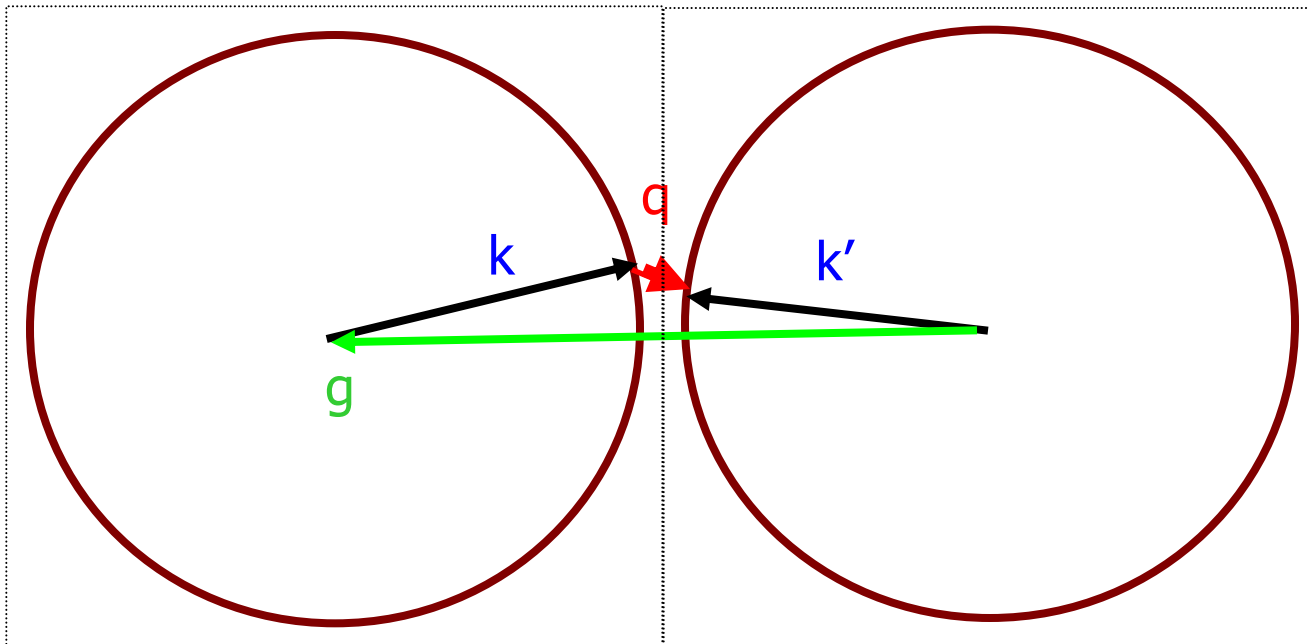
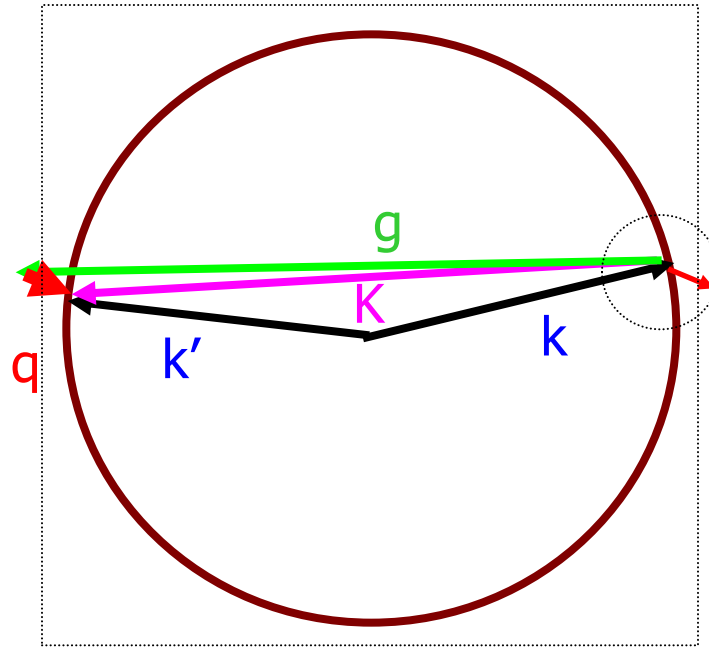
FIG. 9. Shift of the electron distribution due to an applied field.

Direct electron scattering on phonons at low temperatures does not contribute much to resistivity, electrons can not scatter at large angle due to momentum conservation law

The only efficient way: Umklapp processes, scattering with momentum transfer to reciprocal lattice



Umklapp processes



Electron-electron scattering

At low temperatures

$$\rho = \rho_0 + \alpha T^n$$

$n=5$ e-ph

Not very efficient in scattering

In e-e scattering total momentum is conserved

Scattering does not contribute to resistivity

- Umklapp

- Electrons from different bands s-d scattering

$$\sigma = (Ne^2/m)(\alpha\tau_a + \beta\tau_b).$$

Bands a and b, densities α and β

Relaxation times τ_a and τ_b

Mott: τ_a and τ_b are of the same order of magnitude;

α and β can be very different,

High probability of scattering into large DOS band

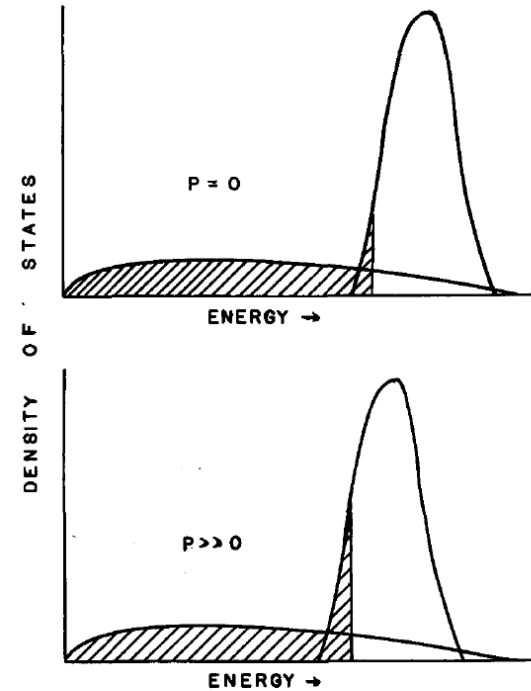


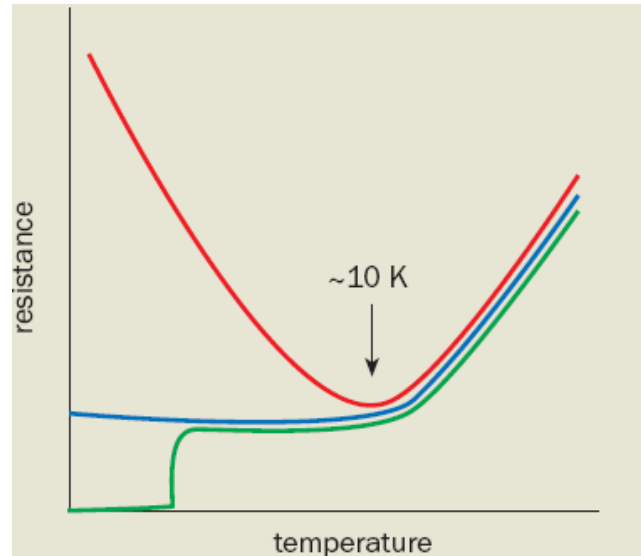
FIG. 14. Occupied electronic states in two overlapping bands at (a) zero pressure, (b) some high pressure (schematic).

$n=2$ e-e

$n=3$ s-d

Kondo effect

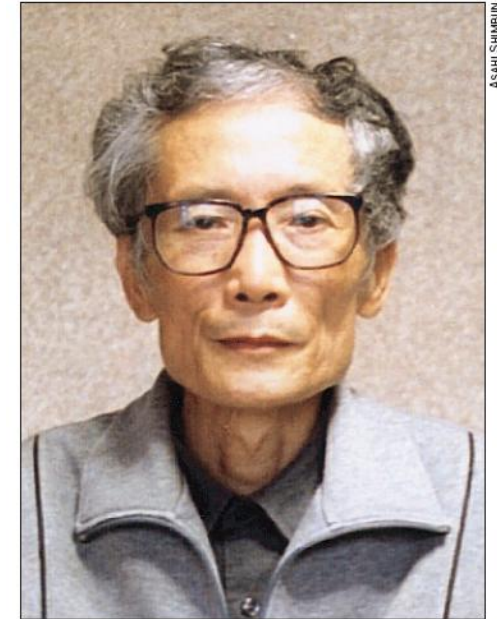
Dilute magnetic
impurities in metals



Interaction of conduction electrons
with magnetic impurities

Characteristic temperature T_K
(varies a lot! From 0.1mK Mn in Au
to 300K V in Au, usually ~ 10 K)

Specific heat per impurity $C \sim T/T_K$
Impurity spins completely screened
at low temperatures



The theory that describes the scattering of electrons from a localized magnetic impurity was initiated by the work of Jun Kondo in 1964

b. 1930

Revival of the Kondo effect

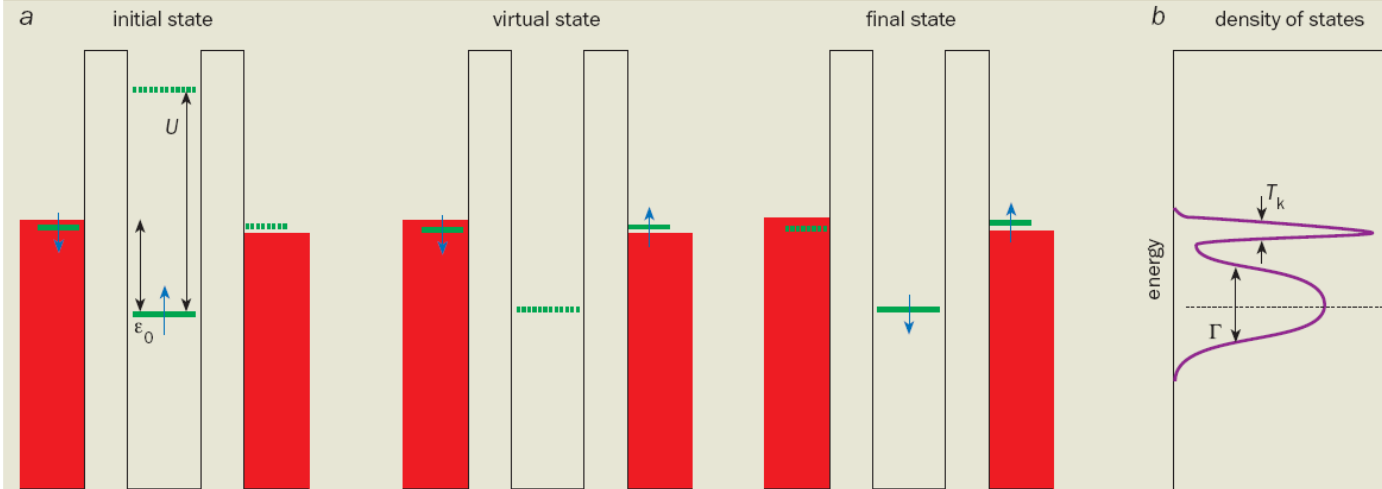
Leo Kouwenhoven and Leonid Glazman

PHYSICS WORLD JANUARY 2001



Anderson impurity model

2 Spin flips



Peak in DOS
Bound to E_F
additional scattering

(a) The Anderson model of a magnetic impurity assumes that it has just one electron level with energy ϵ_0 below the Fermi energy of the metal (red). This level is occupied by one spin-up electron (blue). Adding another electron is prohibited by the Coulomb energy, U , while it would cost at least $|\epsilon_0|$ to remove the electron. Being a quantum particle, the spin-up electron may tunnel out of the impurity site to briefly occupy a classically forbidden “virtual state” outside the impurity, and then be replaced by an electron from the metal. This can effectively “flip” the spin of the impurity. (b) Many such events combine to produce the Kondo effect, which leads to the appearance of an extra resonance at the Fermi energy. Since transport properties, such as conductance, are determined by electrons with energies close to the Fermi level, the extra resonance can dramatically change the conductance.

$\rho/\rho_0 = f(T/T_K)$
universal function of one parameter T_K

$\rho \sim \ln T$

Revival of the Kondo effect
Leo Kouwenhoven and Leonid Glazman

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Magnetic scattering

$\text{Ce}_{1-x}\text{La}_x\text{RhIn}_5$

f-electrons are localized

Same lattice
localized f-electrons
add to scattering

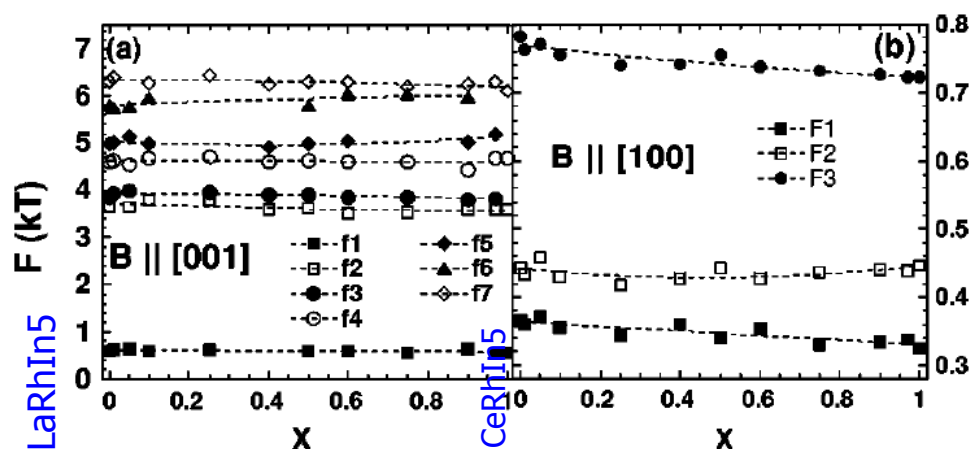


FIG. 2. The principal dHvA frequencies in $\text{Ce}_x\text{La}_{1-x}\text{RhIn}_5$ plotted versus x , (a) for B applied along $[001]$ and (b) for B applied along $[100]$.

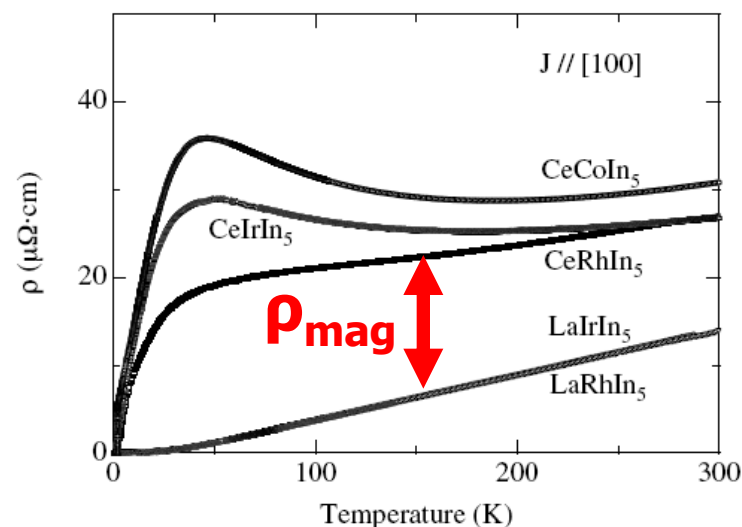
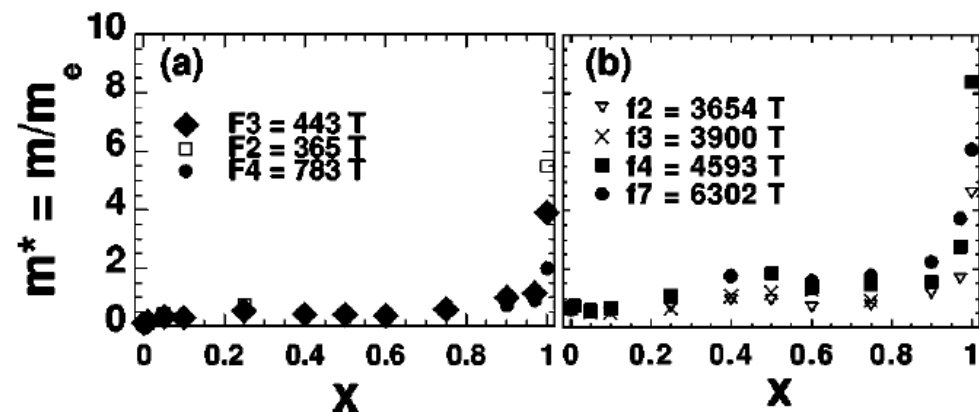
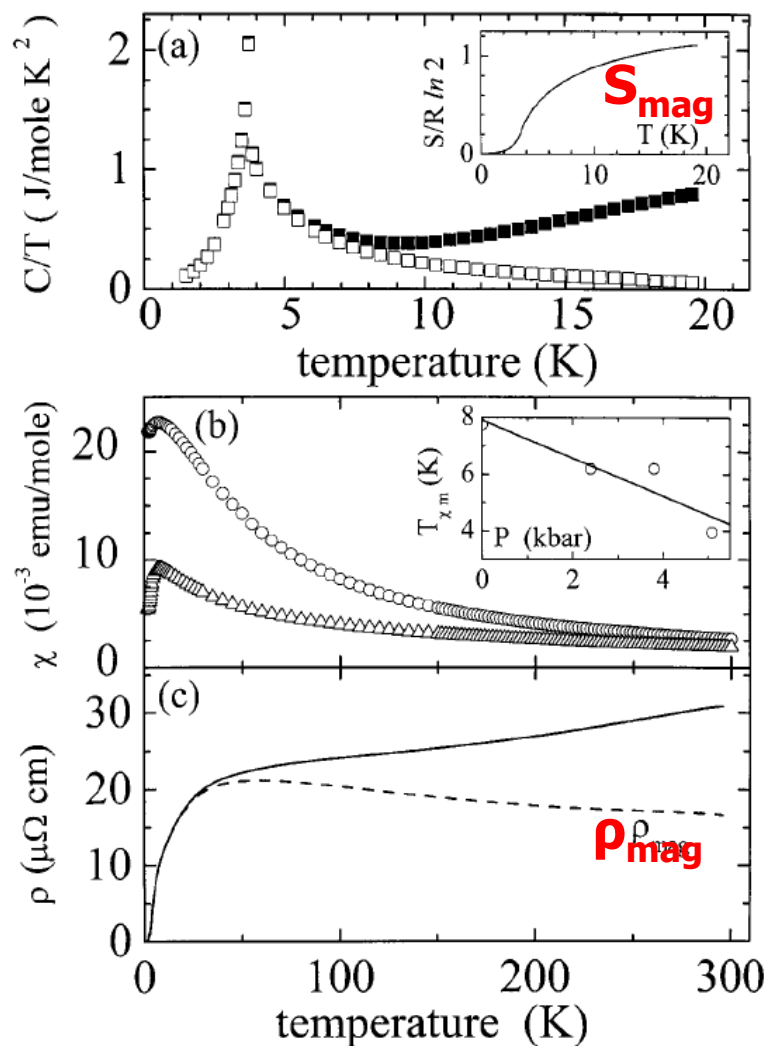


Fig. 3. Temperature dependence of the electrical resistivity in RTIn_5 .



CeRhIn5



$$\rho_{\text{mag}} \sim S_{\text{mag}}$$

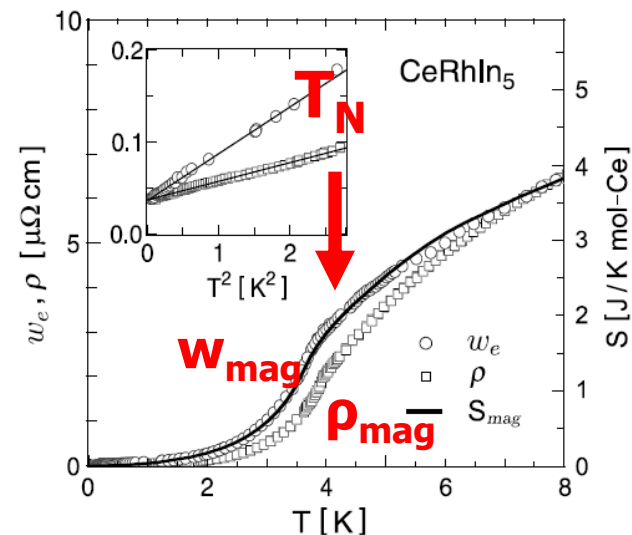


FIG. 2. Electronic thermal resistivity w_e (circles) and electrical resistivity ρ (squares), compared to the magnetic entropy S_{mag} (line) obtained from published specific heat data [12]. Inset: low-temperature data as a function of T^2 . Lines are linear fits.

“q-filtering” removed:
Full scattering directly
proportional to entropy

Density waves

Rudolph Peierls
1907-1995



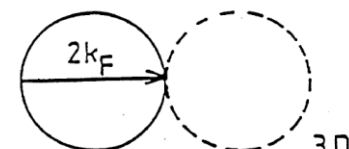
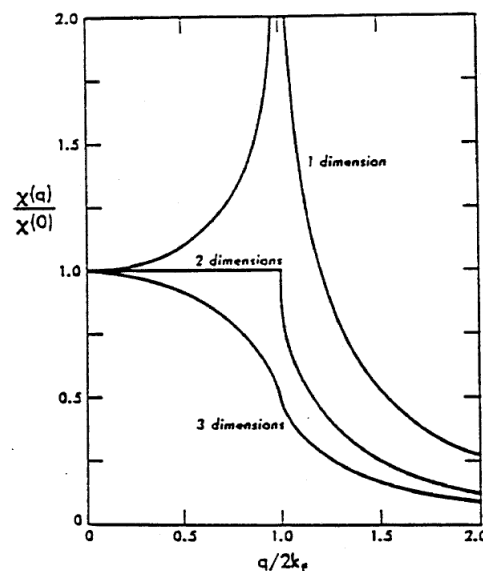
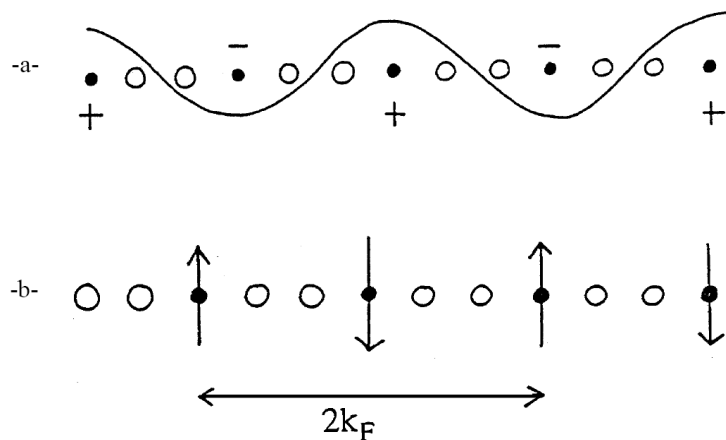
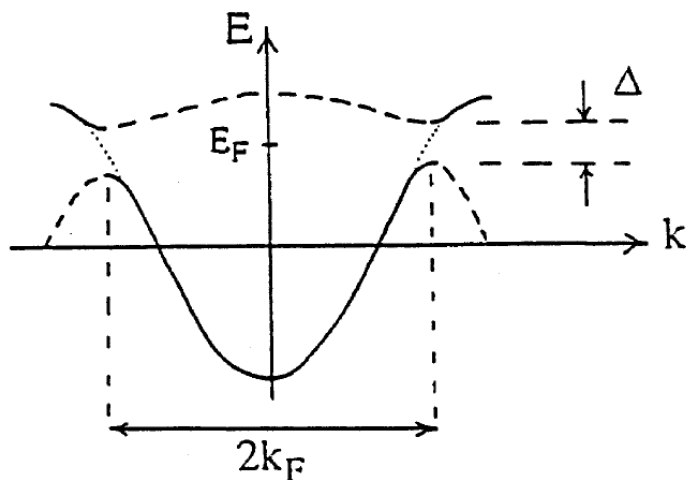
"Frisch-Peierls Memorandum" of March 1940,
proposing a "super-bomb which utilizes the
energy stored in atomic nuclei as a

source of energy."
1957 on request of US government removed
security clearance, suspected of spying

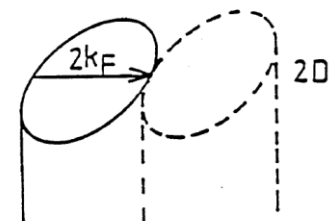
Klaus Fuchs

knighted in 1968

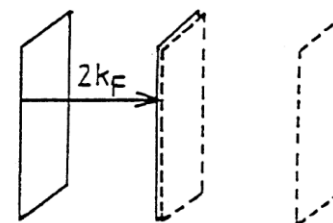
www.history.bham.ac.uk/research/index.htm



3D



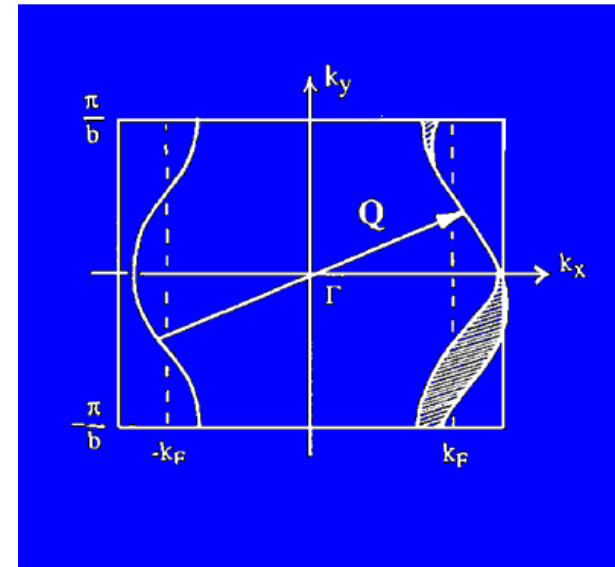
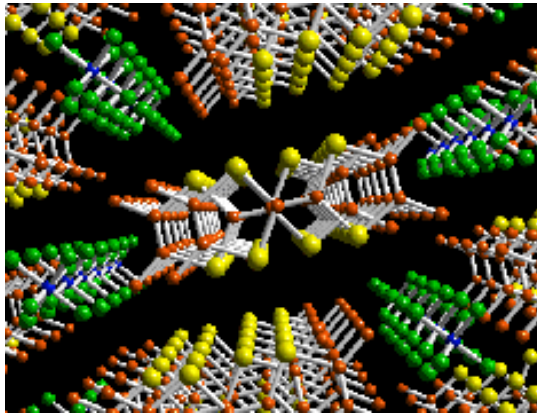
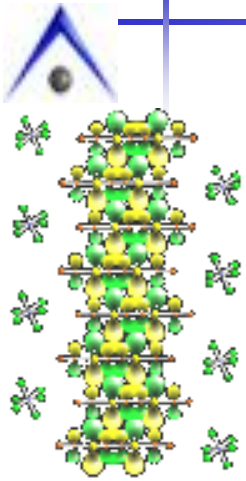
2D



1D

Distortion with $2K_F$ periodicity
Decreases electron energy

BCS-like solution
 $\Delta = 1.75 T_c$



The Fermi surface of a quasi-1D conductor
 has two distinct parts at $+k_F$ and $-k_F$ (here in a cut view).
 Q nests one part to another by translation.

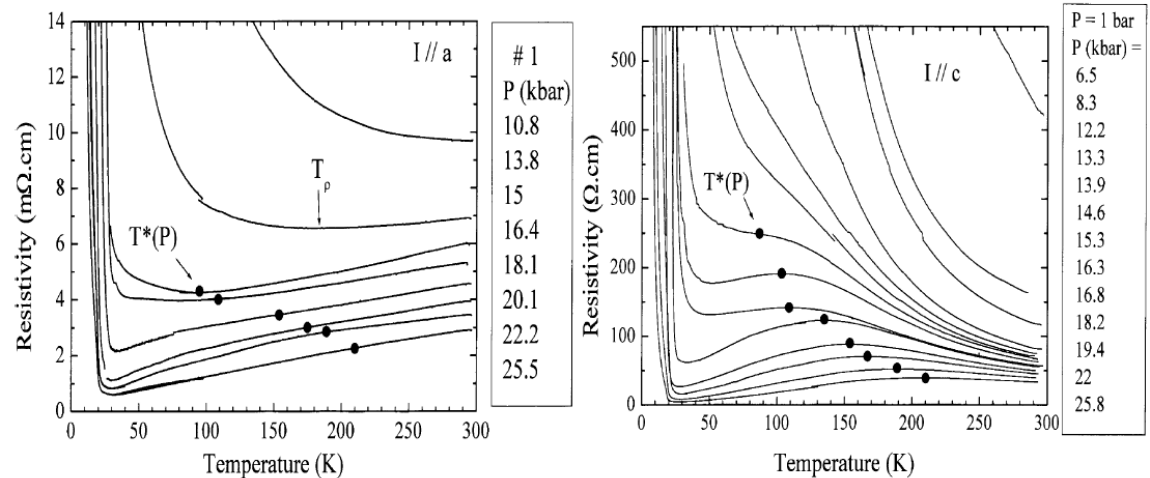
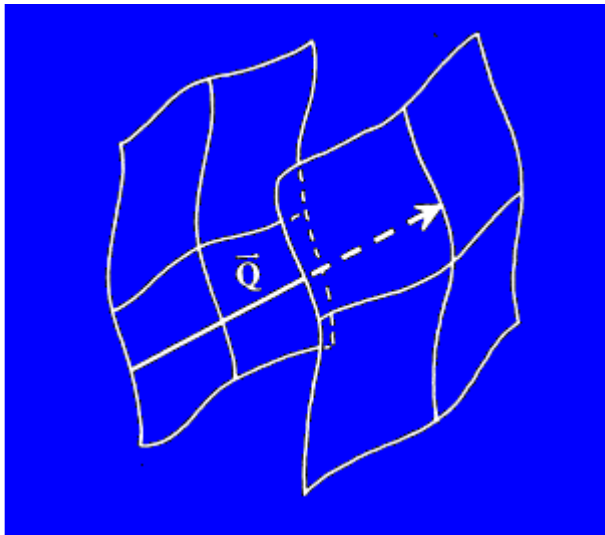


Fig. 2. (TMTTF)₂PF₆ longitudinal (left) and transverse (right) resistances versus temperature at different pressures.¹²⁾

Density waves: increasing dimensionality

Chain

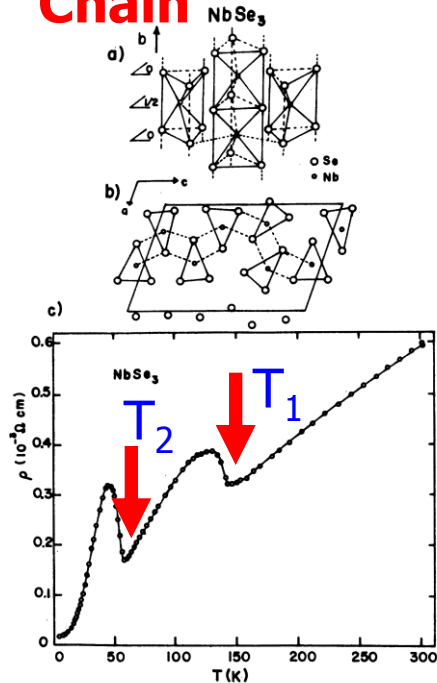


FIG. 1. (a) Crystal structure of NbSe₃ along the *b* axis. The Nb atoms in adjacent chains are displaced by half a lattice spacing along *b* with respect to each other. (b) Crystal structure of NbSe₃ in the *a*-*c* plane. The unit cell, comprised of six prisms and represented by the parallelogram, has the dimensions $a = 10.006 \text{ \AA}$, $b = 3.478 \text{ \AA}$, $c = 15.626 \text{ \AA}$, $\beta = 109.30^\circ$. The Nb-Nb distance along *b* is 3.478 \AA (compared to 2.68 \AA in the metal). In the other directions it varies from 4.45 to 4.25 \AA . The dashed lines connect Nb and Se atoms in the same plane. (c) Resistivity of NbSe₃ vs temperature. The phase transitions at 145 K and 59 K have been tentatively identified with the formation of charge-density waves.

NbSe3

$T_1 = 142 \text{ K}$ ICDW1 $q_1 = (0.245 \pm 0.007) b^*$

$T_2 = 58 \text{ K}$ ICDW2 $q_2 = (0.494 \pm 0.008) a^* + (0.267 \pm 0.007) b^* + 0.50 c^*$

Layer

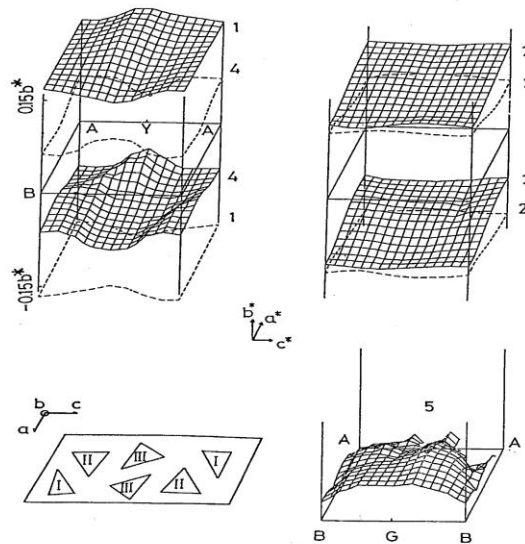


Fig. 1. Fermi surfaces corresponding to each conduction bands and the chain location in a unit cell of NbSe₃. The nesting pairs, 1-4 and 2-3, are pictured in half of the Brillouin zone. Each of the nesting pieces of the Fermi surfaces is drawn in the solid and dashed line. Fermi surface 5 is pictured in quarter of the Brillouin zone.

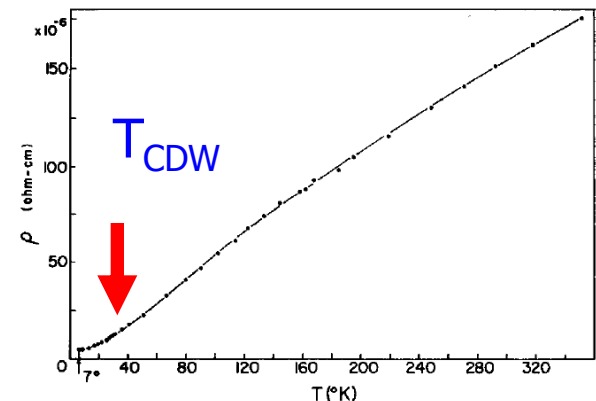


FIG. 1. The resistivity as a function of temperature for NbSe₂.

NbSe2

Density-wave-Like

Anion ordering: big "external" periodic potential with periodicity close to $2K_F$

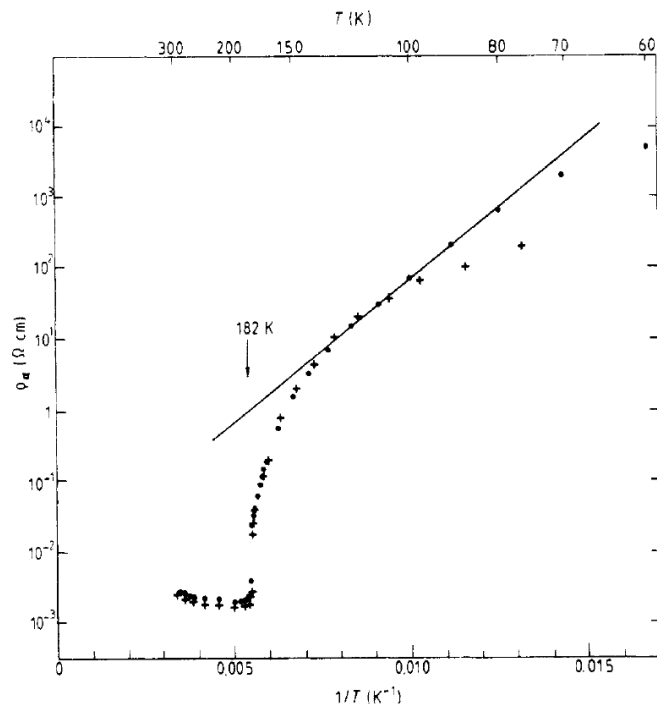


Figure 4. Logarithm of resistivity versus inverse temperature for $(\text{TMTSF})_2\text{ReO}_4$ along the molecular stacks. Results are given for DC (full circles) and for 35 GHz (crosses).

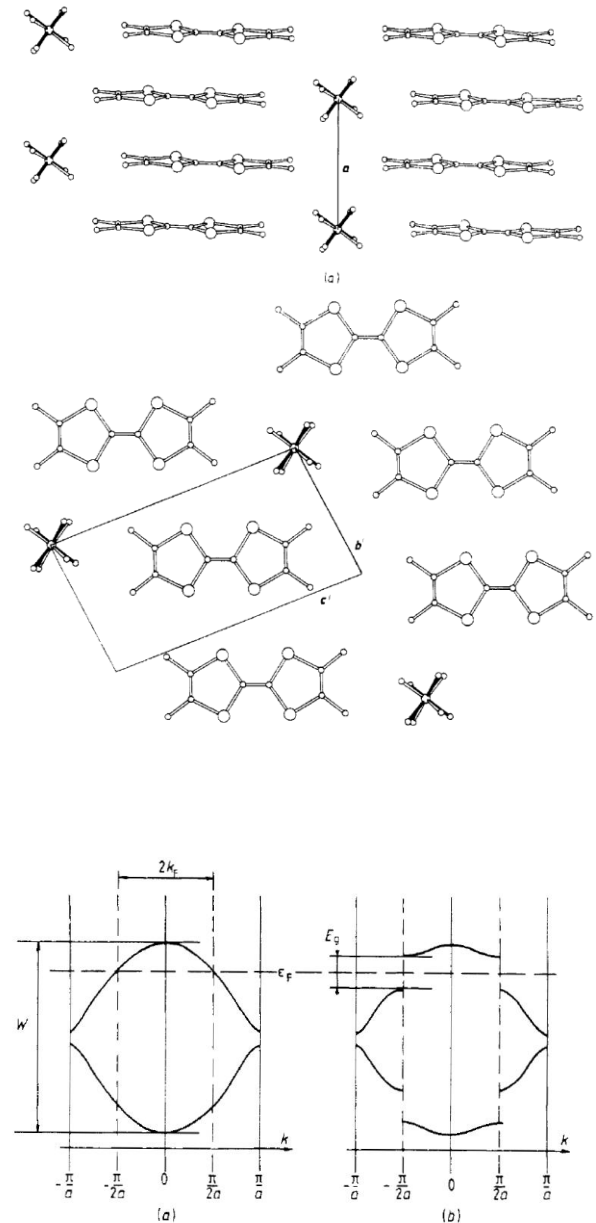


Figure 3. One-electron band structure of $(\text{TMTSF})_2\text{ReO}_4$ shown schematically above (a) and below (b) the metal-insulator transition.

Almost all high- T_c materials have unusual $\rho(T)$ Electrons pair as they scatter!

PHYSICAL REVIEW B

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1 MARCH 1977

Apparent T^2 dependence of the normal-state resistivities and lattice heat capacities of high- T_c superconductors*

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(Received 8 March 1976; revised manuscript received 17 September 1976)

We report new measurements of the normal-state electrical resistances of several high- T_c (~ 20 K) A-15 structure superconductors. It is found that the resistances are linear functions of T^2 from T_c to 40 K. Analysis of available lattice heat-capacity data on the same materials shows that it also varies as T^2 over the same temperature interval. These observations suggest that the T^2 resistivity is due to electron-phonon scattering. This interpretation is supported by the results of a model calculation for the temperature dependence of the resistivity and the lattice-heat capacity of Nb_3Sn .

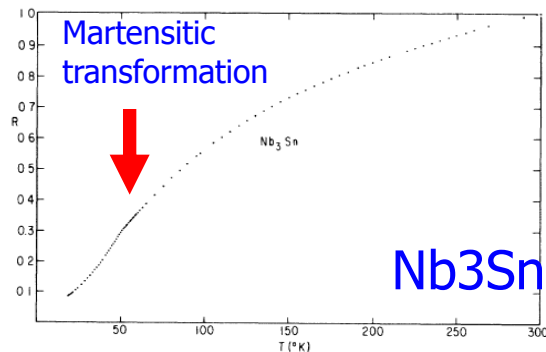


FIG. 1. Resistance of Nb_3Sn from T_c (18.2 K) to 300 K in units where $R(300 \text{ K}) = 1.0$. The kink at 51 K is indicative of a martensitic transformation. Using the data of Woodward and Cody we estimate $\rho(300 \text{ K}) = 75 \mu\Omega \text{ cm}$.

Strong electron-phonon interaction

Unusual $\rho(T)$ dependence: saturation at high T ,
Low T - extended range of T^2 behavior

Correlating with T^2 (rather than T^3)
lattice specific heat

Systematic Evolution of Temperature-Dependent Resistivity in $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

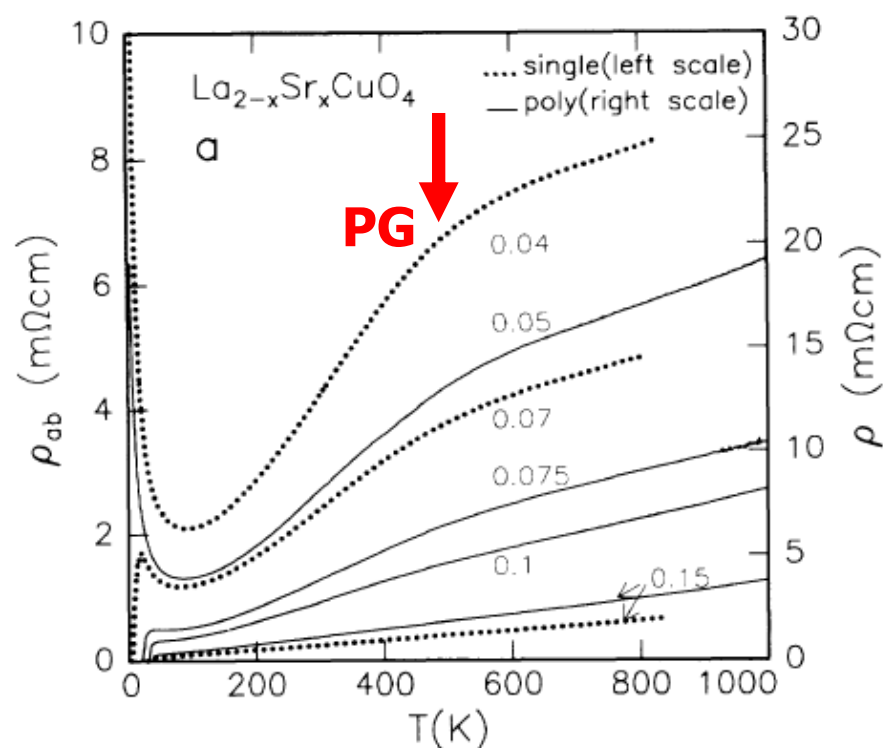
H. Takagi, B. Batlogg, H. L. Kao, J. Kwo, R. J. Cava, J. J. Krajewski, and W. F. Peck, Jr.

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 8 May 1992)

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

$x=0 - 0.15$



$\rho \sim T$ at optimal doping
 $\rho \sim T^n$ at higher dopings

$x=0.1 - 0.35$

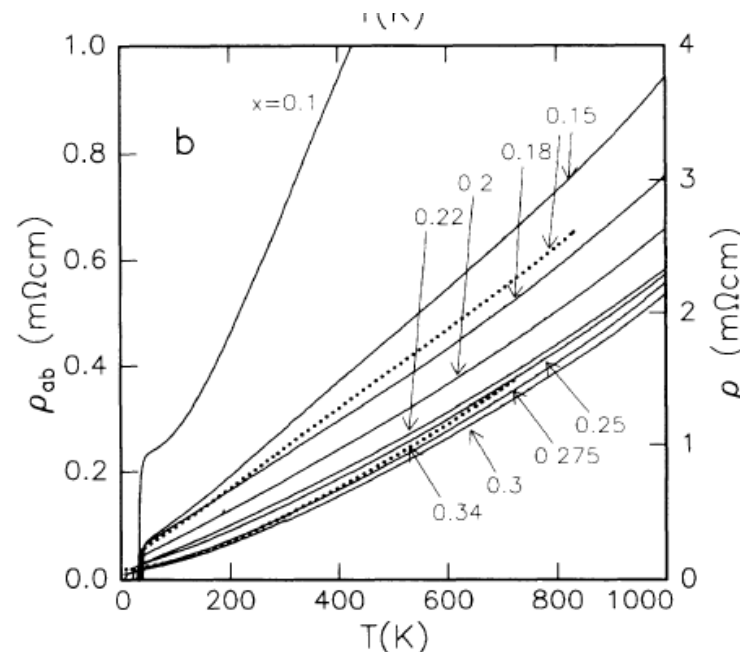


FIG. 1. The temperature dependence of the resistivity for $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$. (a) $0 < x \leq 0.15$, (b) $0.1 \leq x < 0.35$. Dotted lines, the in-plane resistivity (ρ_{ab}) of single-crystal films with (001) orientation; solid lines, the resistivity (ρ) of polycrystalline materials. Note, $\rho_M = (h/e^2)d = 1.7 \text{ m}\Omega \text{ cm}$.



C-axis transport

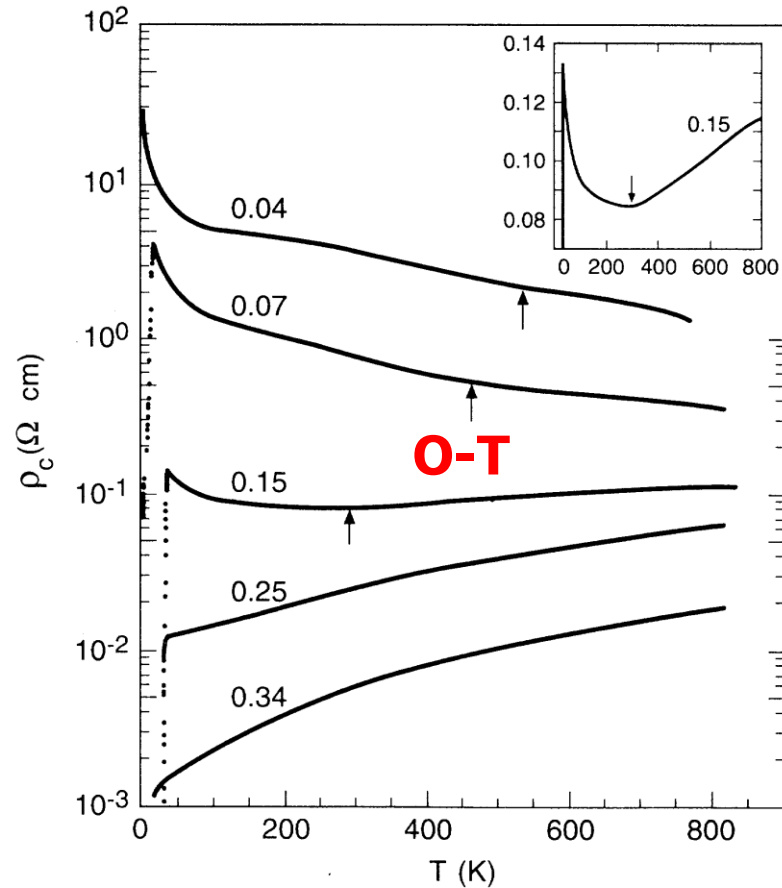


FIG. 3. Semilog plots of ρ_c vs T for $x = 0.04, 0.07, 0.15, 0.25$, and 0.34 . The inset shows the linear plot of ρ_c for $x = 0.15$. The arrows denote the temperature of the orthorhombic-to-tetragonal transformation where a discontinuous change of $d\rho_c/dT$ appears.

Highest T_c heavy fermion CeCoIn₅

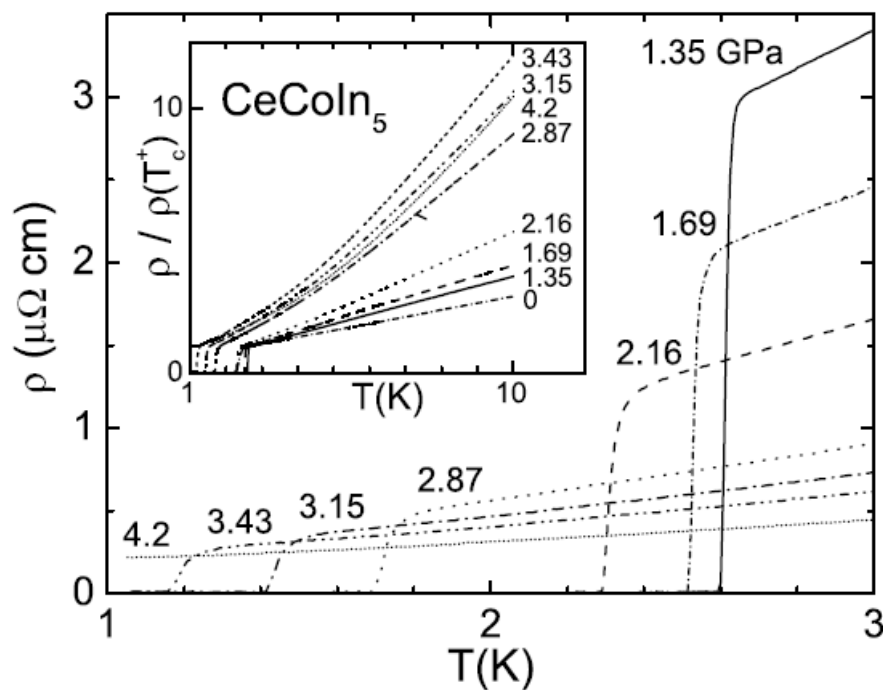


FIG. 1. Effect of pressure on the low-temperature resistivity and superconducting T_c of CeCoIn₅. The inset shows the resistivity ρ normalized by $\rho(T_c^+)$ up to 10 K.

Highest T_c organic superconductors

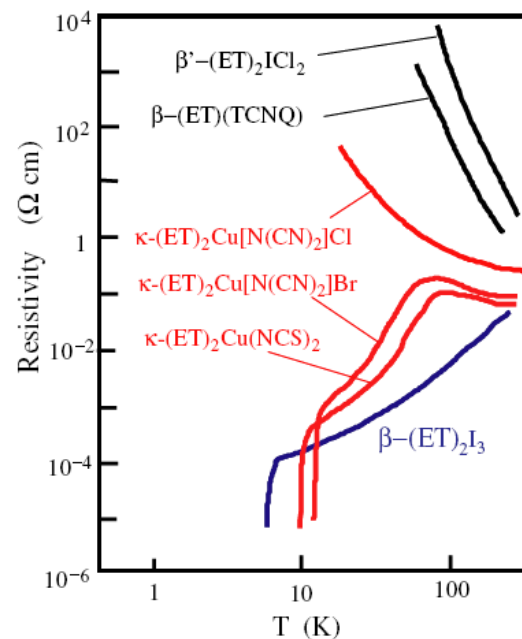


Fig. 2. (Color online) Resistivity behaviors of $\kappa-(\text{ET})_2\text{X}$ and related dimeric compounds.